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SUBJECT: ON THE BEHAVIOR OF JUNCTION TRANSISTORS IN SWITCHING CIRCUITS

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Abstract: In many cases, the observed transient response of a transistor switching circuit does not agree with the transient response obtained by a linear equivalent circuit analysis. This is because a transistor equivalent circuit, composed of ideal, linear or piecewise linear, electrical elements, can only approximate the nonlinear behavior of a junction transistor in a switching circuit. In this paper, a different method of analyzing the transient response of a transistor switching circuit is presented. Essentially, this method involves analyzing an equivalent electrophysical system of the transistor switching circuit in which the transistor is represented by a physical model rather than an electrical equivalent circuit. A qualitative technique is developed for analyzing the electrophysical system with which one can quickly and accurately sketch the salient features of the transient response of any transistor switching circuit. The application of this technique, in obtaining the transient response of a switching circuit, is discussed and several examples, in which it is used, are presented.

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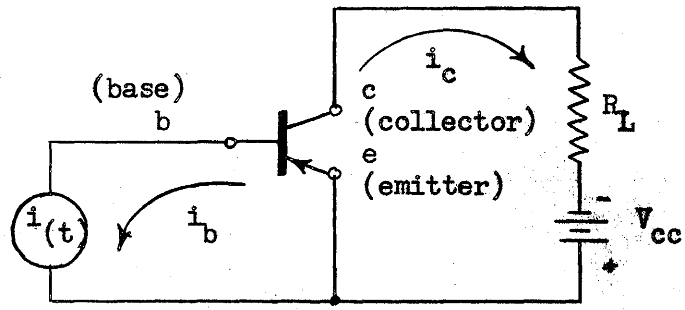
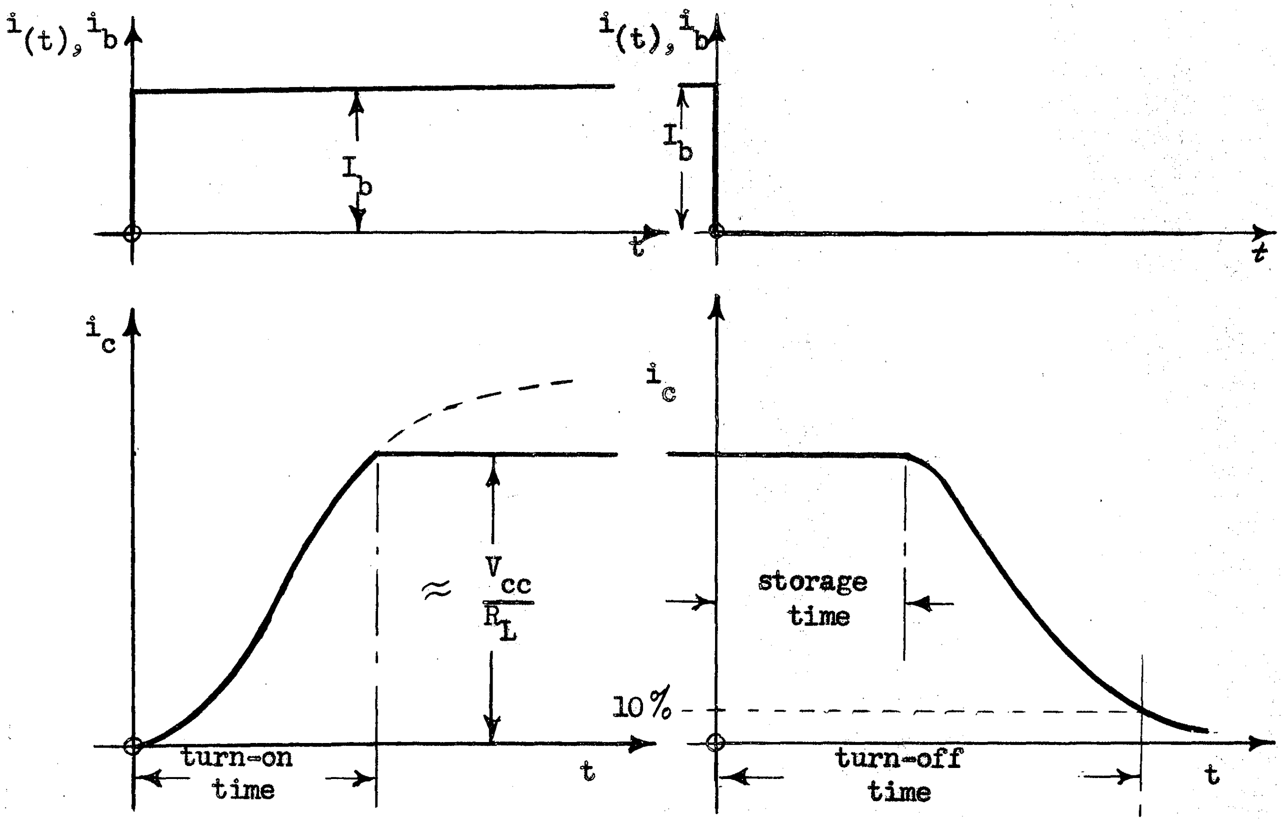
INTRODUCTION

One of the more important properties of a junction transistor is its ability to act as a high-speed switch. A simple circuit in which a junction transistor is used as a switch is shown in Fig. 1 together with the input (base) and output (collector) current waveforms. The operation of this circuit can be described as follows: For zero base current, the impedance between collector and emitter is in the order of 10^6 ohms. Under this "off" condition, essentially no current flows through the load resistor, R_L. However, if a current, I_b, is applied to the base input of the transistor, then the impedance between the emitter and collector is reduced to a level in the order of an ohm. Under this latter "on" condition, a current of approximately V_{cc}/R_L flows through the load resistor, R_L. Thus, by changing the base current from zero to some value I_b, the impedance between collector and emitter can be altered from essentially that of an open circuit to that of a short circuit. In this type of operation, the action of the transistor is similar in many respects to that of an ordinary relay. The transistor, however, can be turned on and off several orders of magnitude faster than the fastest relays. It is this property of high switching speed which makes the transistor superior to the relay for many applications, such as, in high-speed, digital computer circuity where the transistor may be asked to switch on or off in less than 10^{-7} sec. Accordingly, the transient response of the transistor to a turn-on or turn-off step of base current is of considerable interest in the design of high-speed, switching circuits since it is the primary factor involved in determining the time required to switch the transistor on or off.



(a) A Simple Transistor Switching Circuit



(b) Collector Current Transient Response to a "Turn-On" Step of Base Current

(b) Collector Current Transient Response to a "Turn-Off" Step of Base Current

Fig. 1

Transistor Switching Circuit

The transient behavior of the transistor switching circuit in Fig. 1(a) is shown in terms of the collector current transient response in Figs. 1(b) and 1(c). When a turn-on step of base current is applied to the circuit, the collector current (Fig. 1(b)) begins to rise slowly at first, then more rapidly, and finally, slowly again as it approaches its steady-state value. In this type of switching circuit, the steady-state value to which the collector current is headed is designed to be larger than the load current to be switched (V_{cc}/R_L). As a result, when the collector current becomes equal to the load current, the voltage from collector to emitter becomes zero, and the collector current no longer increases. In this condition, the transistor is said to be saturated and it behaves as though the collector is shorted to the emitter. The time required for the collector current to reach its saturation value is called the turn-on time. When a turn-off step of base current is applied to this saturated transistor, the collector current (Fig. 1(c)) does not stop flowing immediately. Instead, the transistor remains in its saturated state for a while and the collector current remains at its saturation value. After a time, called the storage-time, the transistor comes out of saturation and the collector current begins to decay toward zero in the same manner that it increased. Theoretically, the collector current reaches zero at a time, $t = \infty$. Consequently, the turn-off time, to be finite, is defined as the time required for the collector current to reach a value which is 10 per cent of its saturation value.

The use of a linear electrical equivalent circuit in analyzing the transient response of a junction transistor is, in general, limited to the transistor's linear region of operation. Even for this type of operation, the exact shape of the transients cannot be obtained from the equivalent circuit. This limitation comes about because the equivalent circuit, to be useful at all in linear circuit analysis, must be as simple as possible and must be composed of ideal linear, or ideal piece-wise linear, elements. Consequently, in deriving such an equivalent circuit, the actual electrophysical behavior of the transistor is approximated to a considerable degree. The use of a transistor as a switching device, however, involves operating the transistor in regions where its behavior is nonlinear and can only be approximated by a linear equivalent circuit. Thus, the linear equivalent circuits appearing in the literature are not of much use in analyzing the transient behavior of junction transistors in switching circuits.

The purpose of this paper is to present and demonstrate a different method of analyzing switching transients of junction transistors. This method involves analyzing the behavior of an electrophysical model of the transistor under transient conditions. The method, as presented here, is essentially qualitative in that it gives the shape of the switching transients but not, in general, quantitative results, e.g., switching times, rise times, etc. The method can be used to obtain quantitative results but, with the exception of a few special cases, the application of this method to obtain numerical results is cumbersome. However, the ability of the design engineer to obtain a qualitative picture of the transient response of a junction transistor in a given switching circuit is of considerable value in that it enables him to accomplish the following things:

(1) analyze the transient behavior of a given transistor switching circuit,

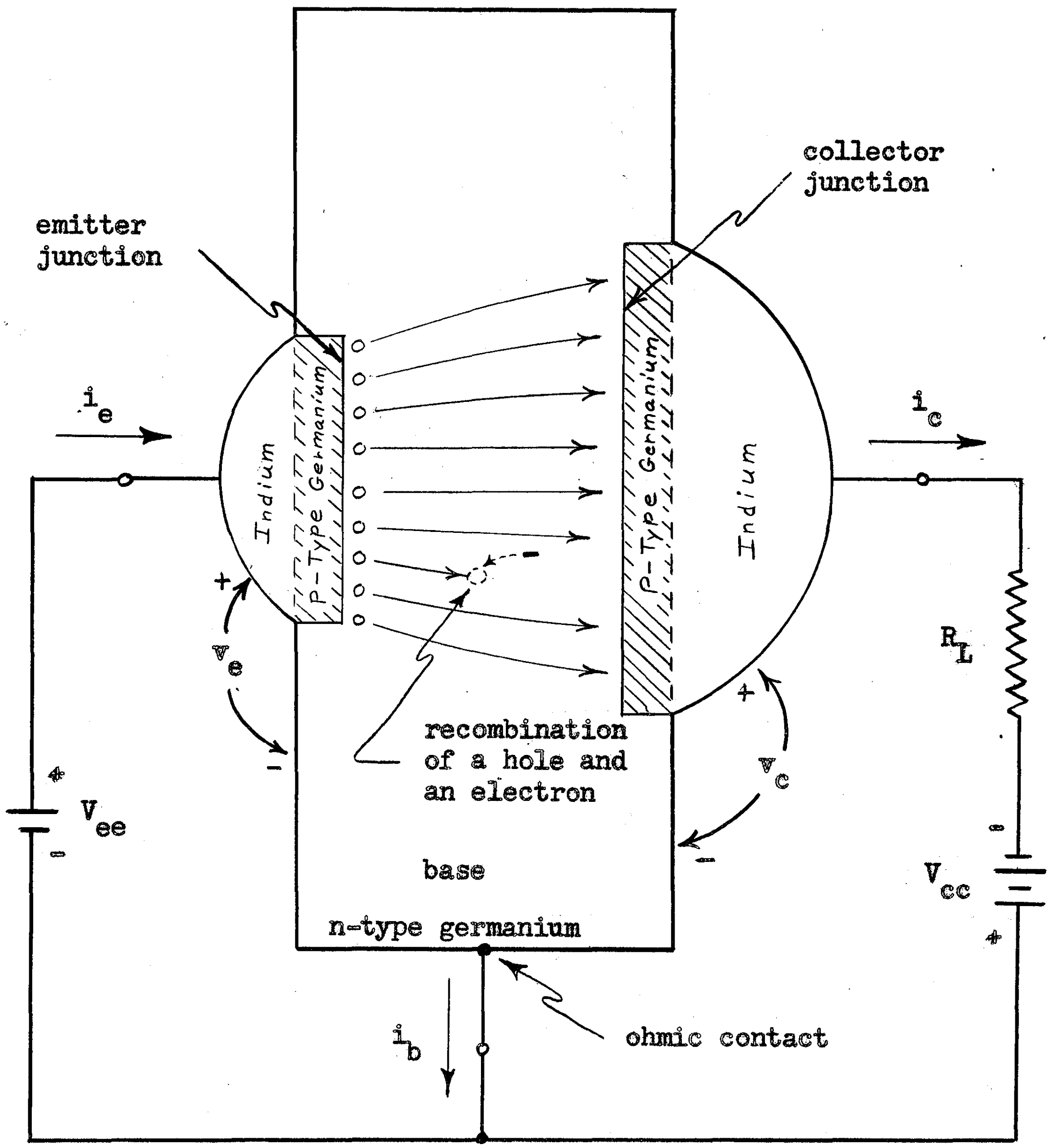
(2) quickly gain a "feel" for the transient performance of the transistor in the switching circuit,

(3) predict the effect of a parameter change on the transient response of the circuit.

In addition, by reducing the particular problem to a limiting case for which analytic solutions can be found, some idea of the values of the switching times involved and their dependence on the various transistor and circuit parameters can be obtained.

BASIC CONSIDERATIONS OF TRANSISTOR ACTION

Fig. 2 shows a cross-section of a p-n-p junction transistor connected as a common-base dc amplifier. For the purposes of this paper, the emitter and collector junctions are considered to be ideal and their only function is to inject or remove minority carriers, i.e., holes in the case of the p-n-p transistor, from the base region. Under normal biasing conditions as shown in Fig. 2, it is the function of the emitter junction to inject holes into the base region and the collector junction to remove them. Quite simply, the operation of the transistor as an amplifying device is as follows: When a small positive voltage, V_{ee}, is applied between the emitter and base of the transistor, holes are injected from the emitter into the base at the emitter junction. If the small ohmic voltage drops that occur in a practical transistor are neglected, then the entire voltage applied between the emitter and base, V_{ee}, and between the collector and base, V_{cc} is dropped across the emitter and collector junctions, respectively. Consequently, the base region of a junction transistor is for all practical purposes field free. This being the case, the holes can only move away from the region of the emitter junction by diffusing away from the region of maximum concentration (emitter junction) in the direction of the negative concentration gradient at a rate which is proportional to the magnitude of this gradient. The desired result



Symbols:
o -- holes
- electrons

Fig. 2

Cross-section of an Ideal P-N-P Alloy Junction Transistor of this diffusion process, in the case of the transistor, is that all the holes injected at the emitter junction will diffuse through the base region to the collector junction where they are immediately absorbed or "collected" from the base by this junction. The ability of the collector junction to remove holes from the base is due to the fact that the electric field which exists at this back-biased junction† causes the collector to appear as a sink for holes.

In order to accomplish the desired effect of having all the holes injected into the base at the emitter diffuse to the collector, the two junctions are placed very close to each other so that the base region separating them is very narrow. Now, since the collector is a sink for holes and the emitter a source, the concentration gradient in the base region between the emitter and collector will be very large and all the holes will diffuse to the collector.

Under steady-state conditions, the collector absorbs holes at the same rate at which they are injected at the emitter, Thus, the current due to hole flow at the collector is equal to and controlled by the current due to hole flow at the emitter. Essentially, then the action of the transistor can be described as a transferring of current in a low impedance circuit (since the emitter junction is forward-biased, the effective impedance of the emitter-base circuit is very low) to a high impedance circuit (since the collector junction is back-biased, the effective impedance of the collector-base circuit is very high). As a result of this current transfer, a power gain is observed to take place through the transistor.

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† The collector junction is said to be back-biased so long as the applied voltage from collector to base, V_{cb}, is negative, i.e., $V_{cb} = i_c R_L - V_{cc} < 0$.

Up to this point in the description of transistor operation, the base current has turned out to be zero since all the emitter current has been assumed to be transferred to the collector circuit. In order to account for the non-zero base current which occurs in any practical transistor, some of the current flowing in the emitter circuit must not be transferred to the collector. The mechanism which is responsible for preventing this total transfer of current is called recombination. Some of the holes in diffusing through the base recombine with an electron and are lost. As a result, not all of the hole current injected at the emitter reaches the collector. The non-zero base current arises because the electrons lost in the recombination process must be replaced by a flow of electrons into the base through the base lead in order to maintain the base region at a constant potential with respect to the emitter.

The rate at which holes and electrons recombine per unit volume at any point in the base region depends on the ratio of the excess hole density, p, at that point, to the average life time of a hole in the base region, τ_p. The base current due to recombination can, thus, be expressed by the equation

$$I_b = q \int_v \frac{p}{\tau_p} \, dv \qquad (1)$$

The diffusion equation

The behavior of injected holes in the base region of a transistor is governed by a diffusion equation of the form,

$$D_p \nabla^2 p - \frac{p}{\tau_p} = \frac{\partial p}{\partial t} \qquad (2)$$

where

$p = p_{(x,y,z,t)}$, the excess hole density at any time and point in the base region,

D_p is the diffusion constant for holes in the base,

and τ_p is the average lifetime of a hole in the base.

Essentially, (2) relates the space rate-of-change of the hole flow (actually, the space rate-of-change of the hole density gradient) and the hole rate-of-decay due to recombination, at any point in the base region, to the time rate-of-change in the hole density at that point. In theory, it is possible to solve (2) for any base geometry and any appropriate set of boundary and initial conditions. Thus, the hole density and its gradient in the base region of the transistor can be obtained as analytic functions of time and position.

In a practical switching transistor, the width of the base region between the emitter and collector is made extremely small compared to the other dimensions of the base region. Consequently, almost all of the holes injected into base at the emitter diffuse in a direction perpendicular to the planes of the emitter and collector. In view of this, the one-dimension form of the diffusion equation can be used to describe the behavior of injected holes in the base region of a switching transistor. For the purposes of this paper, then, the diffusion equation will be considered to be of the form,

$$D_p \frac{\partial^2 p}{\partial x^2} - \frac{p}{\tau_p} = \frac{\partial p}{\partial t} \qquad (3)$$

where $p = p_{x,t}$.

It will be shown that the boundary and initial conditions for (3) are determined by the voltages and currents at the emitter and collector junctions. Thus, for any given input driving function of voltage or current the transient response of the collector current can be obtained by solving (3) for the hole density gradient at the collector since the collector current is proportional to the hole density gradient at this boundary.

Boundary conditions

For the purposes of this paper the analytical solutions of the diffusion equation will be restricted to two types of boundary conditions. The first type is one in which the function itself, p, is prescribed at the boundary. In the second type, the gradient of the function, $\partial p/\partial x$, is prescribed at the boundary. In general, the boundary conditions will be known only in terms of voltages and currents. In order to express these electrical quantities in terms of the hole density and its gradient at the junctions, it is necessary to establish the relationships that exist between these electrical and physical parameters at a junction. These relationships, as obtained by Shockley,† are given by the equations

$$p_{\varepsilon,c} = \begin{cases} p_{no}\left(e^{(q/kT)v_{\varepsilon,c}} - 1\right) & \text{for } v_{\varepsilon,c} \geq 0 \\ 0 & \text{for } v_{\varepsilon,c} \leq 0 \end{cases} \qquad (4)$$

† Shockley, W., et al, "The P-N Junction Transistors," Physical Review, Vol. 83, pp 151-162, July, 1951.

and

(5)

where

p_e, p_c -- are the hole densities in the base region at the emitter and collector junctions, respectively.

V_e, V_c -- are the voltage drops across the emitter and collector junctions, respectively, as measured from emitter to base and from collector to base (see Fig. 2).

i_e, i_c -- are the hole currents at the emitter and collector junctions, respectively.

$\frac{p}{x}_e, \frac{p}{x}_c$ -- are the hole gradients in the base region at the emitter and collector junctions, respectively.

P_{no} -- is the normal equilibrium hole density in the base region.

k -- is Boltsman's constant.

q -- is the electronic charge, and

T -- is the temperature in degrees Kelvin.

<u>An idealized model of the junction transistor</u>

Fig. 3 shows a one-dimensional model of a p-n-p junction transistor based upon the above discussion. The abscissa or x-dimension in the figure represents distance through the base region in a direction perpendicular to the planes of the emitter and collector junctions, the positive direction of x being from right to left. The emitter and collector junctions are represented by the boundaries $x = w$ and $x = 0$, respectively, and are assumed to be ideal step junctions. The ordinate or p-direction represents the excess hole density at any point in the base. In general, p is a function of time, t, and distance, x, which is determined by the diffusion equation as given by

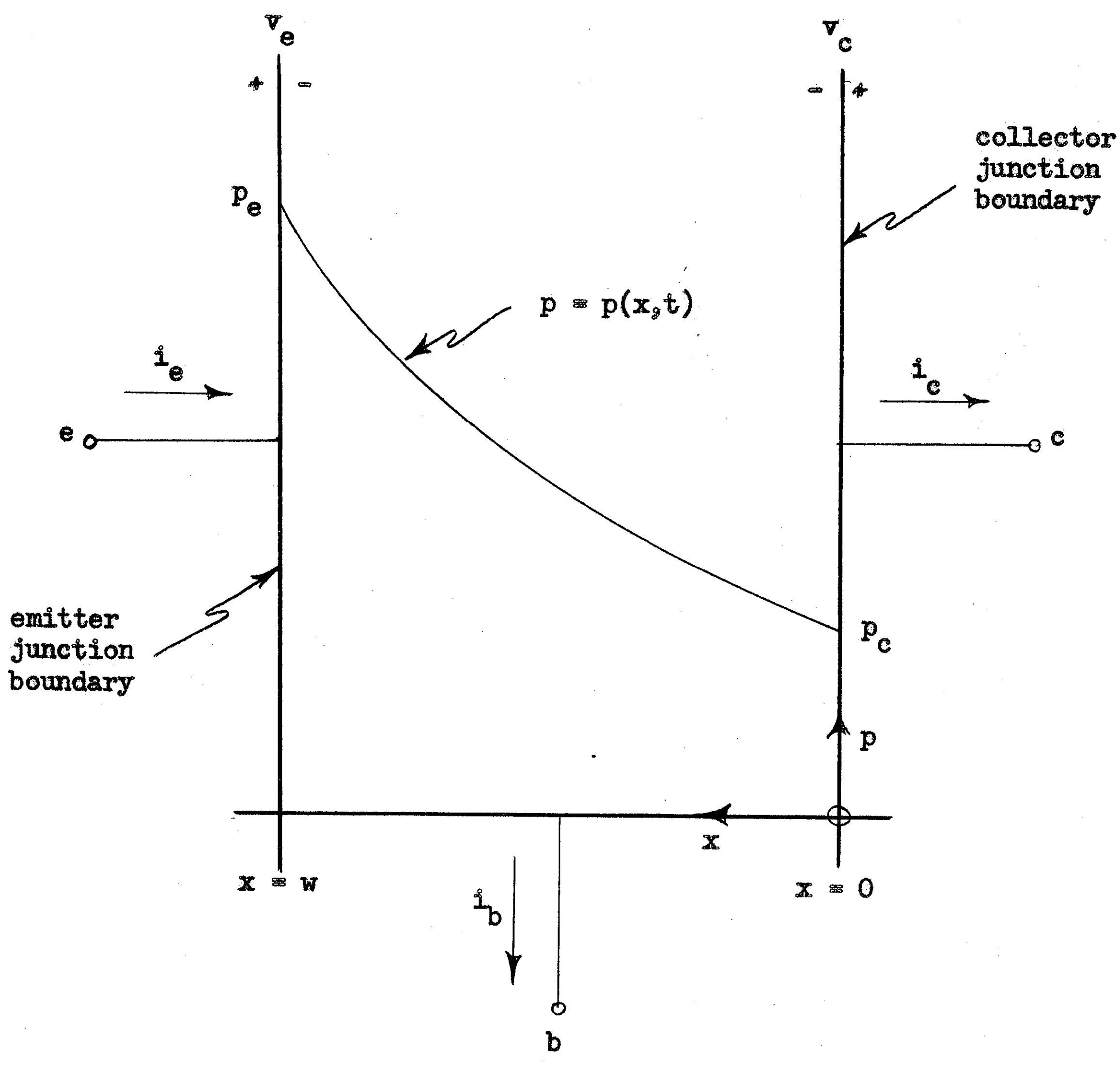


Fig. 3

An Idealized, One-Dimensional Model of a P-N-P Junction Transistor

(3) and the boundary conditions at the emitter and collector junctions as determined from (4) and (5). The hole density at the boundaries of the emitter and collector junctions are defined to be p_e and p_c, respectively. The currents i_e, i_b, and i_c are the emitter, base, and collector currents, respectively, and because of the one-dimensionality of the model, they have the dimensions of current density. The voltages V_e and V_c are the voltage drops across the emitter and collector junction, respectively, and are considered to be positive when the drop occurs in going from the emitter or collector to the base.

ANALYTIC METHODS OF SOLVING THE TIME DEPENDENT DIFFUSION EQUATION

The most common analytic method of solving the particular form of the diffusion equation given by (3) is the method of separation of variables. This method is restricted in the sense that, in general, solutions can be obtained only if the boundary condition can be reduced to a homogeneous set. Fortunately, many of the problems encountered in practice have boundary conditions which satisfy this restriction.

For boundary conditions of the form

$$P_\mathcal{E} \quad \text{or} \quad \left.\frac{\partial P}{\partial x}\right|_\mathcal{E} = K_\mathcal{E} = a \text{ constant} \tag{6}$$

and

$$P_c \quad \text{or} \quad \left.\frac{\partial P}{\partial x}\right|_c = K_c = a \text{ constant}, \tag{7}$$

the general procedure for solving the time dependent diffusion equation as given by (3) is to put it in the form

$$\frac{\partial^2 P}{\partial x^2} - \frac{P}{L_p^2} = \frac{1}{D_p}\frac{\partial P}{\partial t} \qquad (3')$$

where $L_p = \sqrt{D_p \tau_p}$, the diffusion length for holes in the base region.

and let the solution be of the form

$$P(x,t) = P_{ss}(x) - P_T(x,t) \qquad (8)$$

Pss(x) is the steady-state solution of the hole density distribution in the base after the transient effects have become negligible and is obtained by solving the steady-state form of (3') for the boundary conditions (6) and (7). $P_T(x,t)$ represents the transient portion of the solution which, in this case, must approach zero as t increases indefinitely. This transient solution is obtained by solving (3') for the modified (as a result of writing the solution in the form of (8)) and, now, homogeneous boundary conditions

$$P_E \text{ or } \left.\frac{\partial P}{\partial x}\right|_E = 0 \qquad (6')$$

$$P_c \text{ or } \left.\frac{\partial P}{\partial x}\right|_c = 0 \qquad (7')$$

with the initial and final conditions

$$P_T(x,0) = P_{ss}(x) - P(x,0) \qquad (9)$$

and

$$P_T(x,\infty) = 0, \qquad (10)$$

respectively.

The case for which one of the boundary conditions given by (6) and (7) is not a constant, but some function of time, f(t), can be solved by use of the superposition integral of the form,

$$P(x,t) = f(0)\, P_1(x,t) + \int_0^t P_1(x, t-\tau) \frac{df(t)}{dt} d\tau \qquad (11)$$

where $p_1(x,t)$ is the solution of the time dependent diffusion equation when the boundary condition described by f(t) is set to unity.

If f(t) is not differentiable, the superposition integral of the form,

$$P(x,t) = f(t)\, P_1(x,0) + \int_0^t f(\tau) \frac{\partial P_1(x, t-\tau)}{\partial t} d\tau \qquad (12)$$

can be used.

Once the solution for the hole density distribution, P(x,t), is obtained, the desired transient response of the emitter and collector currents can be found by forming the gradient of the hole density, $\partial p/\partial x$, and substituting this expression into (5). A typical example in which the transient response of the emitter and collector currents is obtained by an analytical solution of the time dependent diffusion equation is given in Appendix I. The detailed procedures involved in solving the time dependent diffusion equation for various types of boundary condition can be found in many texts which treat the partial differential equations of mathematical physics[3] or, in particular, the equation of heat conduction.[4]

A QUALITATIVE APPROACH TO THE SOLUTION OF THE DIFFUSION EQUATION

By making use of the characteristic behavior of a quantity governed by the diffusion equation, the solution of this equation can usually be sketched in considerable detail. The transient response of the transistor can then be obtained from the behavior of the hole density and its gradient (slope) at the junction boundaries.

Qualitative solutions for τ_p infinite

When the diffusion equation is written in the form,

$$D_p \frac{\partial^2 p}{\partial x^2} - \frac{p}{\tau_p} = \frac{\partial p}{\partial t}$$

it states that the rate-of-change of hole density at any point in the base is proportional to the sum of the curvature of the hole density distribution, $D_p \partial^2 p/\partial x^2$ and the negative rate-of-decay of the hole density at that point, p/τ_p. In a region where the curvature is large, i.e., $D_p(\partial^2 p/\partial x^2) \gg \frac{p}{\tau_p}$, the diffusion equation reduces to the form

$$D_p \frac{\partial^2 p}{\partial x^2} \doteq \frac{\partial p}{\partial t} \tag{13}$$

Essentially, this equation shows that the rate-of-change of hole density at a point, in a region where the curvature of the hole density distribution is large, is related to the curvature at that point in such a manner that the hole density increases with time if the curvature is positive and decreases with time if the curvature is negative. Furthermore, the speed at which this increase or decrease in hole density takes place, i.e., $\partial p/\partial t$ is proportional to the magnitude of the curvature. Some examples of this behavior of the hole density distribution are shown in Figure 4 for zero curvature and two different values each, of positive and negative curvature. The time interval between t_1 and t_2 is the same in each case and is assumed to be much less than τ_p so that the loss of holes due to recombination, during this interval, is negligible. One characteristic property of the diffusion equation which can be seen from

* Here the term "curvature" is loosely used to mean the second derivative. The actual radius of curvature is the inverse of the quantity: $p''(1+p'^2)^{-3/2}$

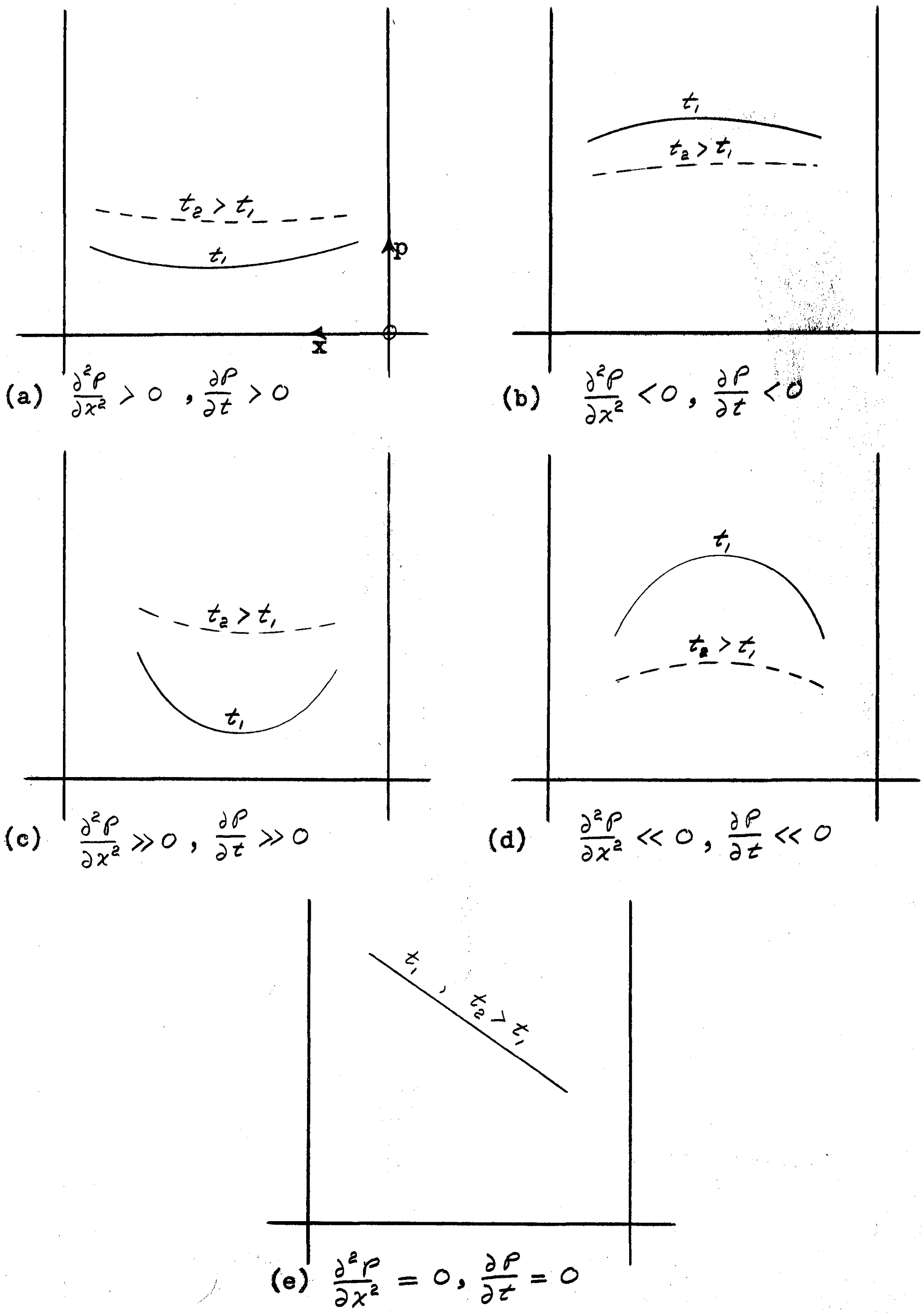


Fig. 4

Effect of Curvature on the Hole Density Distribution

Fig. 4 is that the change in hole density at any given point is in such a direction as to tend to reduce the curvature of the hole density distribution in the region about that point as time increases.

In view of (13), it is apparent that if

$$\frac{\partial^2 p}{\partial x^2} \longrightarrow 0$$

then

$$\frac{\partial p}{\partial t} \longrightarrow 0$$

Therefore, the function, $p(x,t)$, does not tend to overshoot its equilibrium value at any point in the base region, but approaches it monotonically as t increases without limit. Thus we can conclude that the solution of the diffusion equation as given by (13) is not oscillatory in nature.

In many cases, the lifetime of the holes in the base region, τ_p, may have a value sufficiently large that $p(x,t)/\tau_p \approx 0$ for any time t. The diffusion equation is then of the form given by (13), and the behavior of $P(x,t)$ is as described above. The equilibrium distribution of the hole density $P_{ss}(x)$ is given by the steady-state form of the diffusion equation which in this case may be written as

$$\left.\frac{\partial^2 P(x,t)}{\partial x^2}\right|_{t=\infty} = \frac{d^2 P_{ss}(x)}{d x^2} = 0 \qquad (14)$$

By integrating (14) twice with respect to x, the solution for $p_{ss}(x)$ is obtained in the form

$$P_{ss}(x) = c_1 x + c_2 \qquad (15)$$

where the constants of integration, C_1 and C_2, are evaluated from the boundary conditions (6) and (7). Thus, for cases where the hole rate-of-decay due to recombination is negligible ($p(x,t)/\tau_p \approx 0$), the equilibrium solution of (13) yields a straight line hole density distribution as given by (15). A case in which the hole density distribution is governed by (13) is treated in the following example.

Example No. 1, τ_p infinite

Fig. 5a shows a modified form of the simple switching circuit of Fig. 1. In this circuit, a voltage step is applied to the base at t = 0 (by closing the switch) instead of a current step. At the instant the switch is closed, a voltage, V_1, appears across the emitter junction. Thus, from (4) the boundary condition at the emitter is given by

$$p_\varepsilon = p_{no}\left(e^{q/kT\, V_1} - 1\right) = a \text{ constant} \qquad (16)$$

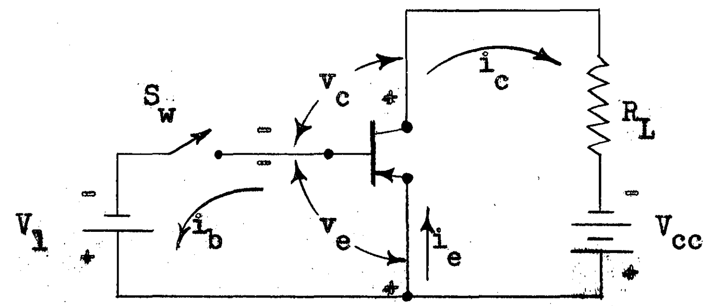
where $V_e = V_1 > 0$.

In obtaining the boundary condition at the collector junction, the magnitude of the voltage step applied to the base, V_1, is assumed to be less than V_{cc}. This implies that the collector junction is back-biased at least for t near zero. From (4) then, the collector boundary conditions is found to be

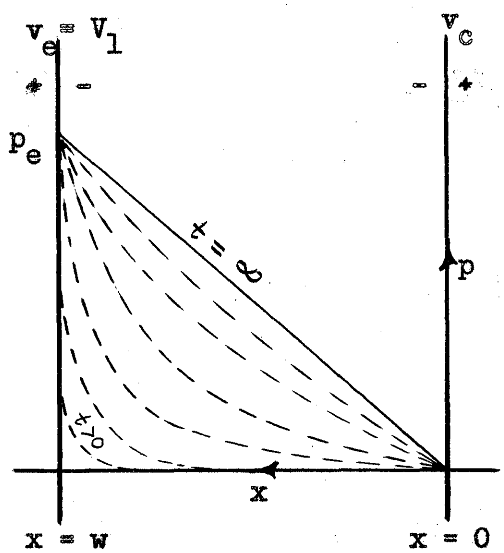
$$p_c = 0 \qquad (17)$$

where $V_c = V_1 + i_c R_L - V_{cc} < 0$ for t near zero

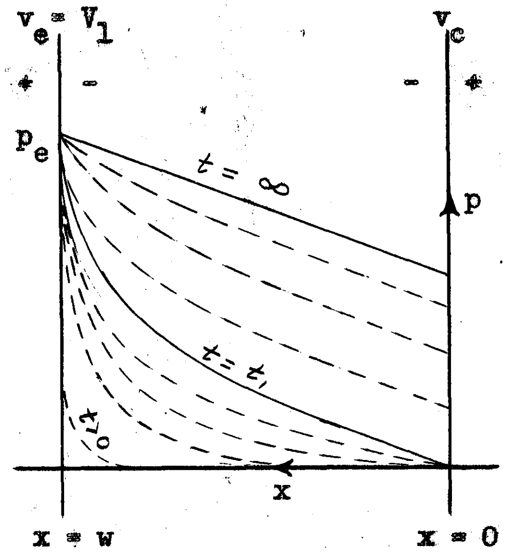
The initial condition, $p_{(x,0)}$, can be seen from (16) and (17) to be a step function of the form



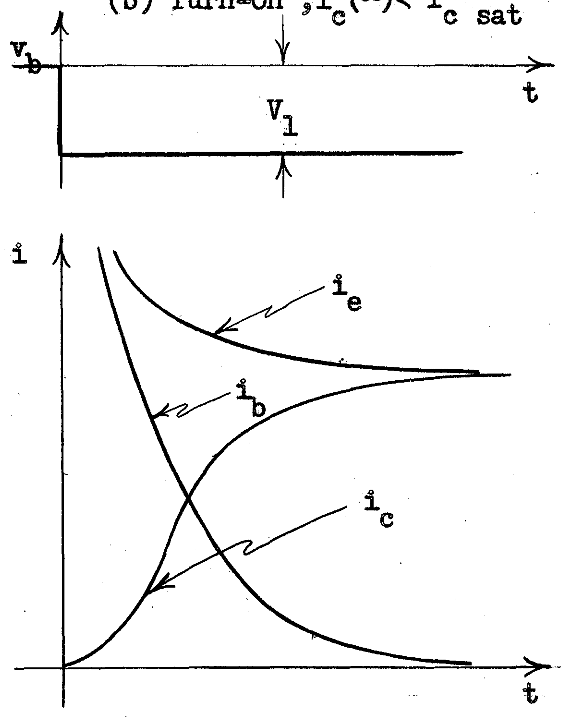
(a) Common Emitter Switching Circuit



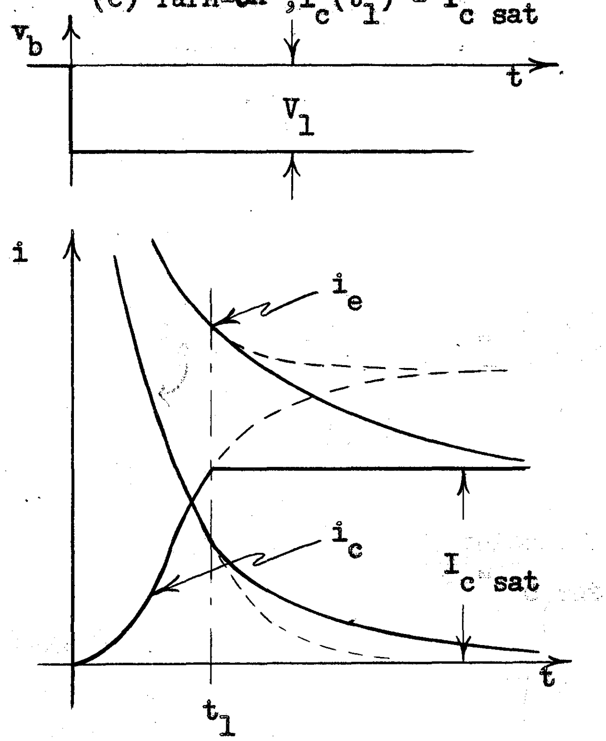
(b) Turn-On , $i_c(\infty) < I_{c\ sat}$



(c) Turn-On , $i_c(t_1) = I_{c\ sat}$



(d) Turn-On Transient Waveforms for i_e, i_b, and i_c; $i_c(\infty) < I_{c\ sat}$



(e) Turn-On Transient Waveforms for i_e, i_b, and i_c; $i_c(t_1) = I_{c\ sat}$

Fig. 5

$$P(x,0) = P_E \left[\mathcal{U}_{-1} (w - x) \right] \tag{18}$$

The curvature of $p_{(x,t)}$ for $t > 0$ is positive as shown in Figs. 5(a) and 5(c) and, consequently, $p_{(x,t)}$ increases with time everywhere in the base except at the junction boundaries where it is clamped by the boundary conditions.

As the current in the collector circuit begins to build up in direct proportion to the gradient (slope) of the hole density at the collector boundary, the solution of $p_{(x,t)}$ can proceed in two different ways depending upon the external parameters of the collector circuit. At some value of collector current defined as $I_{c\ sat}$, the voltage across the back-biased collector junction becomes zero, i.e.,

$$v_c = V_1 - V_{cc} + i_c R_L = 0 \tag{19}$$

when

$$i_c = I_{c\ sat}$$

Now in the sketch of the solution for $p_{(x,t)}$ shown in Fig. 5(b), it has been assumed that

$$\lim_{t \to \infty} i_c(t) < I_{c\ sat} \tag{20}$$

which implies that according to (19)

$$\lim_{t \to \infty} v_c < 0 \tag{21}$$

Consequently, the collector boundary condition given by (17) holds as t increases indefinitely, and the solution of $p_{(x,t)}$, as sketched in Fig. 5(b), proceeds to a straight line equilibrium distribution whose end points are given by (16) and (17). By combining (15), (16), and (17), the equilibrium solution is found to be of the form

$$P_{ss}(x) = P_E \frac{x}{W} \tag{22}$$

If, however, as in the sketch of the solution for $p_{(x,t)}$ shown in Fig. 5(c), it has been assumed that at some time $t = t_1$

$$i_c(t) = i_c(t_1) = I_{c\ sat}, \tag{23}$$

then the collector boundary condition given by (17) no longer holds. For as t becomes greater than t_1, $i_c(t)$ tends to become greater than $I_{c\ sat}$ and V_c becomes greater than zero. Accordingly, for solutions of $p_{(x,t)}$ where $t > t_1$ another boundary condition must be specified at the collector junction. It can be shown that, for $V_c \geqslant 0$, the current in the collector circuit remains nearly constant and is approximately equal to $I_{c\ sat}$. The new boundary condition at the collector junction for $t > t_1$ is then according to (5), of the form

$$\left.\frac{\partial P(x,t)}{\partial x}\right|_c \approx \frac{I_{c\ sat}}{q D_p} = a\ constant \tag{24}$$

In order to see that (24) does hold for $t > t_1$, consider, the equation for the current in the collector circuit when $V_c \geqslant 0$. This equation can be written from Fig. 5(a) in the form

$$i_c = \frac{V_{cc} + (V_E - V_c)}{R_L} \tag{25}$$

An expression for $(V_e - V_c)$, where V_e, $V_c \geqslant 0$, can be obtained from (4) in terms of p_e and p_c in the form

$$V_E - V_C = \frac{kT}{q} \ln\left(\frac{P_E + P_{no}}{P_C + P_{no}}\right) \tag{26}$$

For the circuit of Fig. 5, it can be seen that p_c can vary only within the range $0 \leqslant p_c \leqslant p_e$. When these limiting values of p_c are substituted into (26), it is found that $V_e - V_c$ can only have values in the range $V_e \geqslant (V_e - V_c) \geqslant 0$. The normal operating value of V_e in junction switching transistors is in the order of a .1 volt, while V_{cc} is at least an order of magnitude greater than this. Thus, from (19) and (25), the approximate relation

$$i_c \Big|_{t > t_1} \approx \frac{V_{cc}}{R_L} = I_{c\,sat} \quad (V_c \geqslant 0) \tag{27}$$

where

$$V_{cc} \gg V_e \sim .1 \text{ volt.}$$

can be written which establishes the veracity of (24) as the collector boundary condition for $t > t_1$.

As t increases indefinitely, the solution of $p_{(x,t)}$, as sketched in Fig. 5(c) for $t > t_1$, proceeds to a straight line equilibrium distribution for the boundary condition given by (16) and (24). This equilibrium solution for $P_{(x,t)}$ is found from (15), (16), and (24) to be of the form

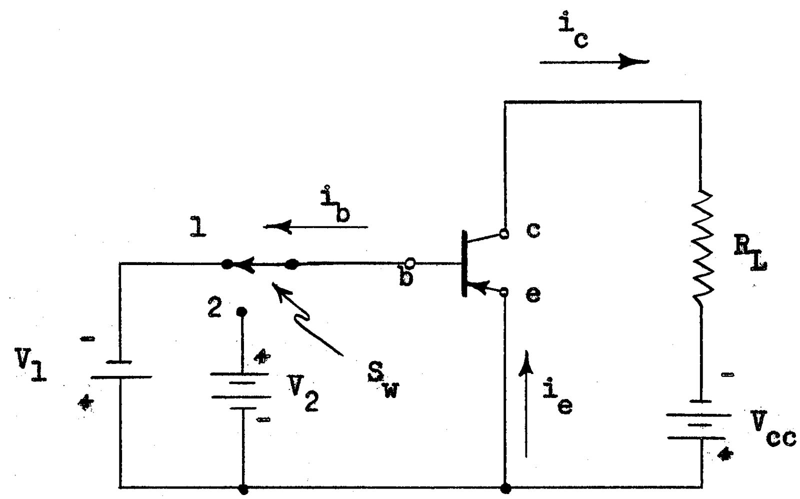
$$P_{ss}(x) = P_E - \frac{I_{c\,sat}}{q D_p}(W - x) \tag{28}$$

The transient response of the emitter and collector currents can be sketched from the behavior of the gradient of the hole density at the emitter and collector junction boundaries, respectively. Fig. 5(d) shows the transient response for $i_e(t)$, $i_b(t)$ and $i_c(t)$ corresponding

to the solution of $p_{(x,t)}$ shown in Fig. 5(b) for the case, $i_c(\infty) < I_{c\ sat}$.
Fig. 5(e) shows the transient response for the same currents corresponding to the solution of $p_{(x,t)}$ shown in Fig. 5(c) for the case, $i_c(t_1) = I_{c\ sat}$. In each case, the base current, $i_b(t)$, is obtained by taking the difference between the emitter and collector currents in accordance with Kirchoff's current law. Since the hole rate-of-decay due to recombination has been assumed to be zero in this example, the equilibrium base current is zero and the equilibrium emitter and collector current are equal. At time $t = 0^+$, the slope of the collector current sketched in Figs. 5(d) and 5(e) is not zero, as might be expected, because the diffusion process is a statistical one and there is a finite probability that a few of the holes injected at the emitter will reach the collector at time $t = 0^+$.

The circuit shown in Fig. 6 is one possible way by which the transistor in the circuit of Fig. 5(a) can be switched off. If V_2 is assumed to be sufficiently large that both the emitter and collector junctions will be back-biased the instant the switch is thrown†, then the problem is identical to the one treated analytically in the Appendix 1. The solution for p as a function of x and t and the resulting transient responses for the emitter and collector current as obtained in the Appendix assumes that the hole density distribution in each case, $i_c(\infty) < I_{c\ sat}$ and $i_c(t_1) = I_{c\ sat}$, has reached the equilibrium distributions as given by (22) and (28), respectively.

† In practice, the emitter and collector junction cannot be back-biased instantaneously for in order to do so infinite currents must occur in the emitter and collector circuits at the first instant and this would require V_2 to be infinite. However, a short time after V_2 is applied to the base, the junctions will become back-biased and the solutions for the currents obtained in the Appendix will become valid.



A Simple Scheme for Turning-Off a Transistor Switching Circuit

Fig. 6

Qualitative solution for τ_p finite

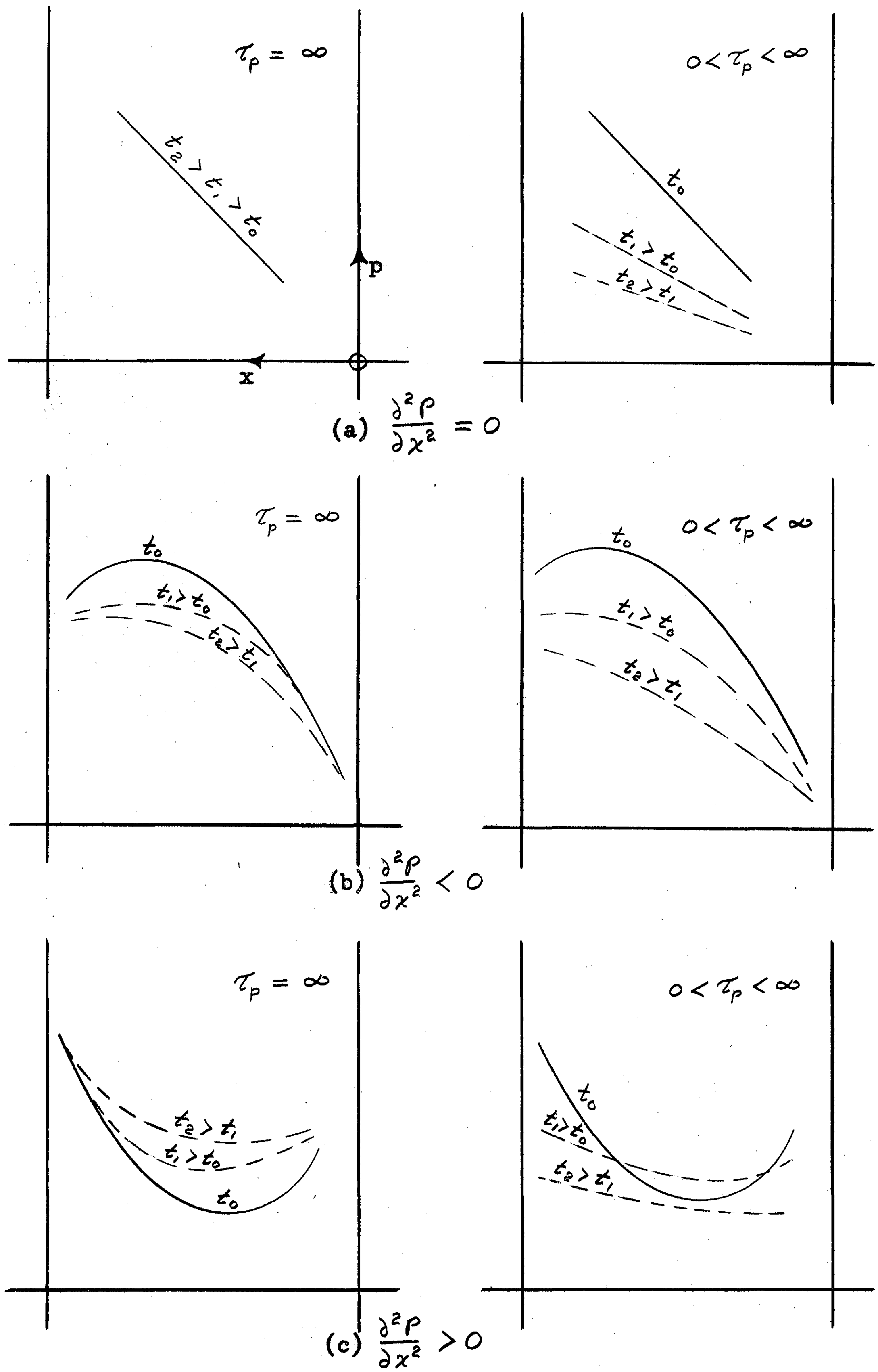
In many cases, it is necessary to take into account the hole rate-of-decay due to recombination in obtaining the solution to the transient response of a switching transistor from the diffusion equation. One reason why this is necessary may be that the hole lifetime in the base, τ_p, is low and the term, p/τ_p, is no longer negligible in comparison with the term involving the curvature of p. This is especially true as the solution for p approaches equilibrium. An even more important reason for taking recombination into account in the diffusion equation exists for practical cases where the base current is required to flow in the transistor during equilibrium, e.g., the circuit shown in Fig. 1. When recombination is neglected as in the case of the problem of Fig. 5, the equilibrium base current is zero. However, when recombination is taken into account, an equilibrium base current must necessarily exist in order to replace the electrons lost in the base region. For the cases just sited and similar ones where recombination cannot be neglected, the transient response of the transistor is governed by the form of the diffusion equation given in (3).

The effect of the recombination term, p/τ_p, in the diffusion equation, on the behavior of $p_{(x,t)}$ is shown in Fig. 7 for several different distributions of the hole density. In addition, a comparison is made for each type of distribution in the figure between the case in which τ_p is infinite (left-hand column in Fig. 7) and a case in which τ_p has some finite value (right-hand column in Fig. 7).

A straight line distribution of p is shown in Fig. 7(a) for which (3) reduces to

$$-\frac{p}{\tau_p} = \frac{\partial p}{\partial t} \tag{29}$$

Thus, for the case in which τ_p is infinite, the distribution remains constant in time since $\partial p/\partial t$ is zero, according to (29). For the case in which τ_p is finite, however, the rate of change of the hole density is negative everywhere in χ, according to (29), and the hole density distribution is seen to decay with time. It is apparent from (29) that, in this latter case, the hole density will decay more rapidly at a point where p is large than at a point where it is small as shown in Fig. 7(a). As a result of this, it can be seen that the recombination term, p/τ_p, in the diffusion equation tends to cause the hole density distribution to flatten out as well as decay with time. In addition, as the hole density distribution decays with time, the over-all rate of decay,



Combined Effects of Curvature and Recombination on the Hole Density Distribution

Fig. 7

according to (29), also decreases. Consequently, the over-all change in the hole density at any point is greater during the interval of time, $t_1 - t_o$, than it is during a later but equal interval of time, $t_2 - t_1$.

In Fig. 7(b) the curvature of the hole density distribution is negative and the hole density decays with time. The rate-of-decay for the case in which τ_p is finite is greater than for the case in which τ_p is infinite because the curvature and the recombination process simultaneously act to cause the hole density to decay in the former case whereas only the curvature causes the hole density to decay in the latter case.

The behavior of the hole density distribution with a positive curvature in a region where τ_p is finite is somewhat ambiguous. According to (3), the two effects, positive curvature and recombination, oppose each other. Therefore, the direction in which the hole density will go at any given time and position depends on whether the curvature or the recombination term dominate the left-hand side of (3) at the time and position specified. Fig. 7(c) shows a hole density distribution whose curvature at time, t_o, varies with distance from essentially zero curvature to some relatively large positive value. In the region of zero curvature, the recombination effect predominates and the hole density decays with time. In the region of large positive curvature, it has been assumed that, initially, effect of positive curvature predominates and, in the time interval, $t_1 - t_o$, the hole density builds up in this region. However, by the end of this initial interval of time, the curvature is assumed to have decreased to such an extent that it no longer dominates the recombination effect. As a result, during the next interval of time, $t_2 - t_1$, the hole density decays. The over-all change in the hole density distribution as a consequence of the above assumptions is shown at the times, t_1 and t_2 in Fig. 7(a). In contrast, if τ_p is infinite, then the hole density will increase with time wherever its curvature is positive and will remain constant wherever its curvature is zero.

The general form of the equilibrium diffusion equation, when τ_p is finite, is obtained by setting the right hand side of (3) to zero. Thus,

$$\frac{d^2 p}{dx^2} - \frac{p}{L_p^2} = 0 \qquad (30)$$

where $L_p = \sqrt{D_p \tau_p}$

The equilibrium hole density distribution is, then, the solution of (30) which can be written in the general form

$$p_{ss}(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p} \qquad (31)$$

where C_1 and C_2 are determined by the equilibrium boundary conditions. For non-zero solutions of $p_{ss}(x)$, the equilibrium hole density distribution is seen, from (30) and (31), to be a transcendental function whose curvature is always positive.

The positive curvature of an equilibrium hole density distribution is indicative of an equilibrium base current. Ordinarily, the hole rate-of-decay due to recombination, p/τ_p, which gives rise to the base current, would cause the hole density everywhere in the base to decay toward zero. However, under equilibrium conditions, this tendency of the hole density to decay is exactly counterbalanced at each point in the base by a tendency of the hole density to increase due to the positive curvature of the distribution at each of these points. Because of this equality between the curvature and the hole rate-of-decay, the equilibrium base current can be expressed in terms of the curvature of the hole density, rather that the hole rate-of-decay as given in (1), by eliminating P/τ_p from (1) and (30). Thus,

$$i_b = q D_p \int_0^W \frac{d^2 P_{ss}(x)}{dx^2} \, dx \tag{32}$$

By carrying out the intergration, it is found that

$$i_b = q D_p \left\{ \left. \frac{d P_{ss}(x)}{dx} \right|_E - \left. \frac{d P_{ss}(x)}{dx} \right|_C \right\} \tag{33}$$

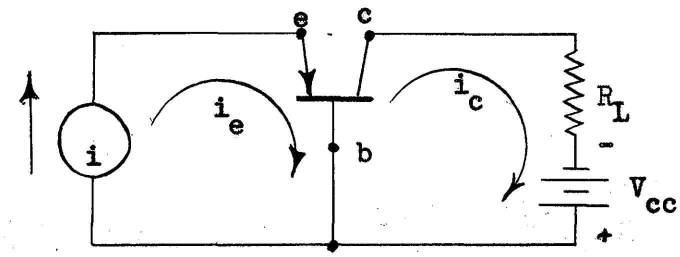
and the equilibrium base current is seen to be directly proportional to the difference in the equilibrium hole density gradients at the emitter and collector boundaries. If the hole density gradients in (33) are replaced by their respective currents at the junctions in accordance with (5), then (33) reduces to the form,

$$i_b = i_E - i_C \tag{34}$$

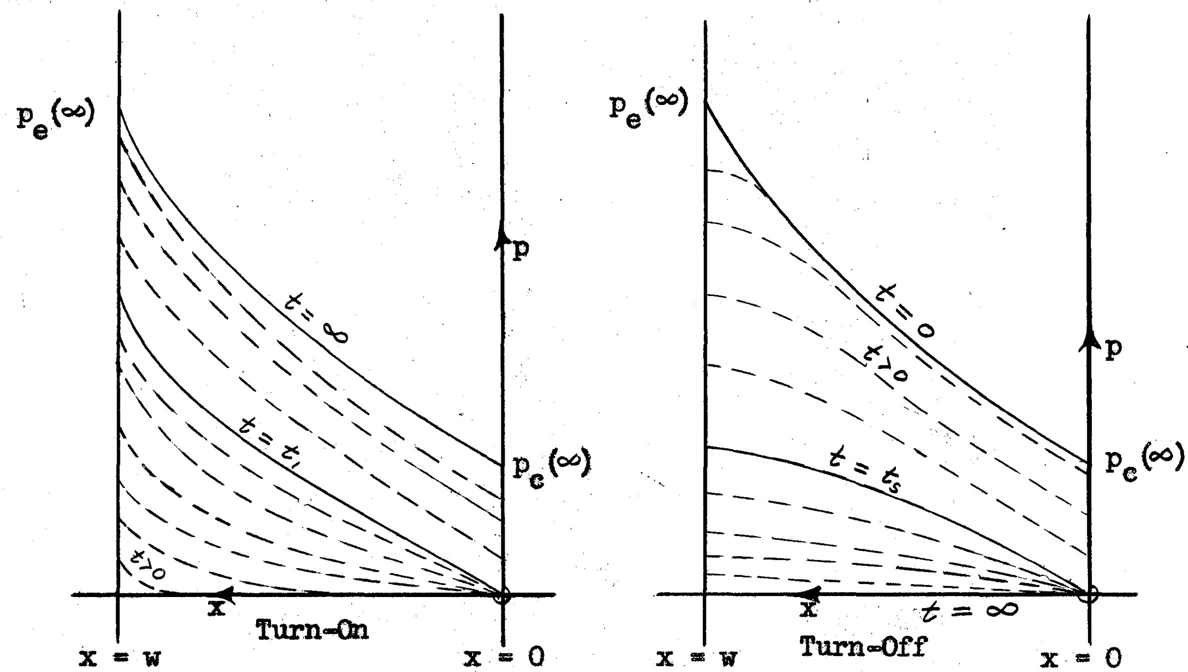
which is Kirchoff's current law for the transistor.

Example #2, τ_p finite

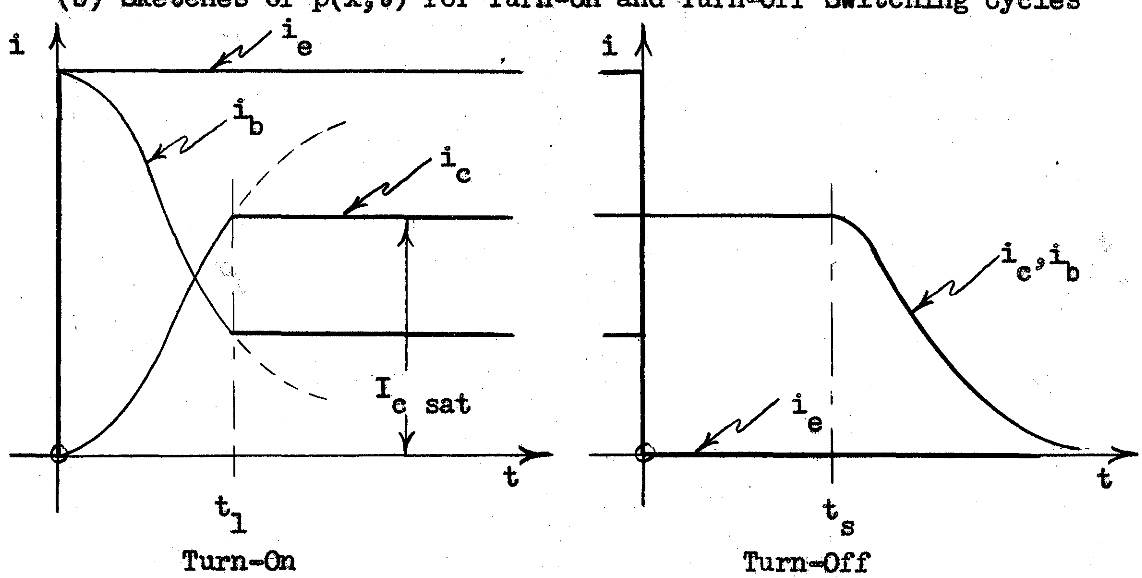
An example of a case in which the effects of recombination on the hole density distribution must be taken into account in the diffusion equation is shown in Fig. 8. The switching circuit shown in Fig. 8(a) is similar in operation to the circuit of Fig. 1 with the exception that the emitter current is used to switch the transistor on and off instead of the base current. The turn-on phase of the transistor's transient response is, thus, initiated by applying a positive step of emitter current at some time, $t = 0$.



(a) A Common Base Switching Circuit



(b) Sketches of p(x,t) for Turn-On and Turn-Off Switching Cycles



(c) Sketches of Transient Waveforms for Emitter, Base, and Collector Currents

Fig. 8

The boundry condition at the collector junction behaves in the same manner as described in the previous example. For $0 \leq t \leq t_1$, the collector current is less than the saturation current $I_{c\ sat}$ *, and the boundry condition is given by (17), i.e.,

$$p_c = 0 \qquad (0 \leq t \leq t_1)$$

For $t > t_1$ **, the collector current is approximately equal to the saturation current and the boundry condition is then given by (24), i.e.,

$$\left.\frac{\partial p(x,t)}{\partial x}\right|_c \approx \frac{I_{c\ sat}}{q D_p} = a\ constant \qquad (t > t_1)$$

The emitter boundry condition is determined by the applied step of emitter current, I_ε, since the hole density gradient is related to I_ε by (5). This boundry condition is, therefore, seen to be of the form

$$\left.\frac{\partial p(x,t)}{\partial x}\right|_\varepsilon = \frac{I_\varepsilon}{q D_p} = a\ constant \qquad (35)$$

* For the definition of $I_{c\ sat}$, see (17).

** In this example it has been assumed that the emitter current, I_ε, is of sufficient magnitude ($I_\varepsilon > I_{c\ sat}$) that the collector current, i_c, reaches the value $I_{c\ sat}$ in a finite amount of time, $t = t_1$.

In order to determine the initial condition of the hole density distribution, $p(x,0)$, it has been assumed that, prior to the time, $t = 0$, the hole density distribution is zero everywhere in the base region. In view of this and the boundary conditions (17) and (35), $p(x,0)$ is zero for all values of x. However, its gradient is a step function of the form,

$$\frac{d\,P(x,0)}{dx} = \frac{I_E}{8\,D_p}\left[u_{-1}\,(w-x)\right] \tag{36}$$

The left-hand drawing in Fig. 8(b) shows a qualitative sketch of the behavior of the hole density distribution in the base as determined from the diffusion equation, (3), and the boundary and initial conditions, (17), (24), (35) and (36) described above. At the instant the positive step of emitter current is applied, $t = 0$, a positive hole density gradient appears in the base at the emitter boundary. The curvature of the hole density becomes infinitely positive at this point and the hole density begins to increase very rapidly in the base region near the emitter boundary. During this phase of the transient response, the actual amount of the hole density in the base is very small and the effect of the hole rate-of-decay due to recombination is negligible.

As the hole density increases and spreads throughout the base region, the curvature begins to decrease and the hole rate-of-decay begins to increase. This causes the rate at which the hole density at any point in the base is building up to decrease. As a result, the hole density distribution tends to approach, monotonically, an equilibrium solution of the form of (31) for the boundary conditions, (17) and (32).

At some time, $t = t_1$, the collector current reaches its saturation value, $I_{c\ sat}$. The collector boundary condition changes, at this time, to that given by (24) in which the hole density gradient is specified at the collector rather than the hole density itself. The hole density distribution now continues to increase at a slower and slower rate in the same manner as before but toward a new equilibrium distribution determined (31), (32) and (24).

It should be pointed out that the magnitude of the new equilibrium hole density, everywhere in the base region, is greater than the magnitude of the original, equilibrium hole density toward which $p(x,t)$ was headed prior to the time, $t = t_1$. This can be seen when it is realized that, in clamping the collector current, i_c , to some value, $I_{c\ sat}$, the resulting equilibrium base current is larger than the original equilibrium base current which would exist if i_c were allowed to reach its normal equilibrium value for the original boundary condition, $P_c = 0$. For, according to the relation between the equilibrium base current and hole density as given by (1), this larger value of base current can only be obtained if the magnitude of the equilibrium hole density is, in general, increased everywhere in the base region.

The transient response of the emitter and collector currents shown in the left hand drawing of Fig. 8(c) are obtained from the behavior of the hole density gradient at the emitter and collector boundaries as sketched in the left hand drawing of Fig. 8(b). The transient response of the collector current, in this case, is seen to behave in a manner similar to the collector current response in the switching circuit of Fig. 1. The response of the emitter current is, of course, a step function. The transient behavior of the base current is found by taking the difference between the emitter and collector currents in accordance with Kirchoff's current law (see equations (33) and (34).

The transistor switch of Fig. 8(a) is turned off by reducing the emitter current to zero. At some time $t = 0^*$, then, the emitter is essentially open-circuited and holes can neither flow in nor out of the base through the emitter junction, i.e., $i_E = 0$. The emitter boundary conditon for $t > 0$ is, then, according to (5) of the form

$$\left.\frac{\partial P(x,t)}{\partial x}\right|_E = 0 \tag{37}$$

The initial condition of the hole density distribution, p(x,0), for this turn-off phase of the transistor's transient response, is given by the equilibrium distribution obtained for the turn-on phase described above. In view of the fact that $p(x,0) > 0$ at the collector boundary, the voltage across the collector junction, ν_c , is greater than zero at the time, $t = 0$. The positive collector voltage cannot change instantaneously, since the collector current must remain finite at all times in view of the circuit conditions. Therefore, it can be said that ν_c remains positive for some finite amount of time t_s after $t = 0$. Because of this and (27), the collector boundary condition, during the time $0 \leq t < t_s$, is given by (24), i.e.

$$\left.\frac{\partial P(x,t)}{\partial x}\right|_c = \frac{I_{c\,sat}}{q D_p} = a \text{ constant} \quad (0 \leq t < t_s)$$

---

* At this time, the hole density distribution is assumed to have reached its "on" equilibrium state as shown in the left hand drawing of Fig. 8(b).

The behavior of the hole density distribution during this time is governed by the diffusion equation, (3), and the boundary conditions, (24) and (37), and is sketched in the right hand drawing of Fig. 8(b). It can be argued from purely physical reasoning why the hole density must decay to zero for the existing circuit conditions by noting that the emitter is no longer a source for holes while the collector junction and the recombination mechanism still act as quasi-sinks for the holes which still exist in the base region. However, the actual manner in which the hole density distribution decays with time can only be obtained from the diffusion equation and the boundary and initial conditions.

At time, $t = 0^+$, the distribution of the hole density in the base region, $w > x \geq 0$, is an equilibrium one, i.e., the positive curvature of $p(x,0)$ prevents the hole density anywhere in this region from decaying by recombination. Initially, then, the hole density distribution tends to remain constant in this region.

This situation is not true at the emitter boundary of the base region ($x = w$), however. According to the initial form of the hole density distribution, $p(x,0)$, the gradient of $p(x,0)$ near the emitter approaches the emitter boundary condition given by (35), i.e., at $t = 0^+$

$$\lim_{x \to w} \frac{\partial P(x,t)}{\partial x} = \frac{I_E}{q D_p} > 0$$

However, at this same time, the gradient at the emitter boundary ($x = w$) must be zero as required by the emitter boundary condition given by (37). This abrupt change in the initial hole density gradient, in going from a point in the base region near the emitter boundary, where the gradient is positive, to the emitter boundary, where the gradient is zero, requires that the initial hole density at $x = w$ have an infinitely large negative curvature. Consequently the hole density at the emitter boundary begins to decay at $t = 0^+$.

At first, the hole rate-of-decay at the emitter boundary proceeds at a very rapid rate because of the large negative curvature in that region of the base. Meanwhile, the decrease in the hole density near the emitter causes the positive curvature of the hole density farther out in the base region to decrease. As a result the hole density there begins to decay due to recombination. Eventually, all of the positive curvature disappears from the hole density distribution and is replaced by a negative curvature that is required if the gradient is to be positive at the collector boundry, as given by (24), and zero at the emitter boundary as given by (37). In this latter form, the hole density distribution decays throughout the base region under the influence of both negative curvature and recombination.

During the time, $0 \leq t < t_s$, the behavior of the hole density in the region of the collector boundary is seen, initially, to remain constant because its positive curvature in that region is in equilibrium with the recombination process. For $t > 0$, the positive curvature begins to decrease and the hole density starts to decay slowly, at first, speeding up as the curvature becomes less positive and more negative. All during this phase of the turn-off transient response, the collector current and, consequently, the hole density gradient at the collector boundary remain constant. This is known as the storage phase of the turn-off time during which the holes "stored" in the base region are drawn out through the collector at a constant rate.

At the end of the storage time, $t = t_s$, the hole density at the collector will have decayed to zero. Because the hole density in the base cannot go negative, the positive gradient at collector as given by (24) can no longer be maintained at this high level. The gradient and, therefore, the collector current must decrease as a result, causing the voltage across the collector junction to go negative. The collector boundary condition is, then, that of a back biased junction ($\nu_c < 0$) as given by (17), i.e.,

$$p_c = 0 \qquad\qquad (t \geqslant t_s)$$

As a result of this change in the collector boundry condition, the negative curvature is no longer necessary since the hole density gradient at the collector is no longer fixed. Consequently, during the time, $t > t_s$, the negative curvature decreases and the hole density distribution tends to approach a straight line which becomes flatter and flatter as the distribution approaches its zero equilibrium condition (see Fig. 7(a) and 7(b). The hole density rate-of-decay continually slows down during this phase since the negative curvature and hole rate-of-decay due to recombination are both approaching zero as time, t, increases indefinitely. The equilibrium, zero hole density distribution is reached, therefore, only as t approaches infinity.

The transient behavior of the emitter and collector currents are shown in the right hand drawing of Fig. 8(c) as sketch from the behavior of the hole density gradients at the junction boundries. The transient response of the collector current is seen to be the same as that described for the switching circuit of Fig. 1. The emitter current transient is, of course, a step function, and is zero for time, $t > 0$. The base current transient is seen to be identical to the collector current transient as required by Kirchoff's current law (see (33) and (34)).

CONCLUSION

Through the use of the qualitative technique of solving the diffusion equation that has been described in this paper, one can determine the large signal transient response of a junction transistor quite accurately in a fairly simple and rapid manner. At first, this method would appear to give only the shape of the transient response without reference to time. This statement, however, is not quite true. A relative time scale can be obtained by this method from the rate at which the hole density distribution increases or decreases, as the case may be. This rate-of-change of the hole density distribution is principly dependent upon the magnitude and sign of the curvature of the distribution which usually can be obtained quite accurately from a sketch of the solution.

Unfortunately, an absolute time scale is lacking in this technique. However, the absolute values of the switching times involved in the various phases of the transient response can be obtained, for a particular circuit under consideration, by observing the output response of the transistor with an oscilloscope. In addition to this, the absolute values of the switching times can usually be obtained from an analytic solution of the transient response of the transistor for either the actual boundary condition involved, or for a limiting case of the class to which the particular problem belongs. When this analytic method is used, not only, can the switching times be determined by evaluating the analytic solution neumerically, but also, their relation to the various transistor papameters involved is established.

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Attachments:

Appendix A

Fig.A2-SA-48860-G
Fig.A3-SA-48861-G
Fig.A4-SA-48862-G
Fig.A5-SA-48863-G

Charles T. Kirk Jr.

Charles T. Kirk, Jr.

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APPENDIX A

An Example of an Analytic Solution to the Transient Response of a Transistor Switching Circuit

The transistor switching circuit for which an analytic solution of its transient response is to be found is shown in Fig. 6. With the switch, S_W, in position 1, the switching circuit is assumed to be "on" and a current, $I_{c\ sat} \approx V_{cc}/R_L$, exists in the collector circuit. The transistor is assumed to have been in this "on" state long enough, prior to some time, $t = 0$, for the hole density distribution to have reached equilibrium as shown in Fig. A1. $P_E(0)$ and $P_c(0)$, in the figure, are the initial equilibrium values of the hole density, in the base, at the emitter and collector junction boundaries, respectively, when S_W is in position 1. The hole lifetime in the base, τ_p, is assumed to have a value sufficiently large that $P(x,t)/\tau_p \approx 0$ for any time, t. The diffusion equation (3), can therefore be written in the form

$$\frac{\partial^2 P(x,t)}{\partial x^2} = \frac{1}{D_p} \frac{\partial P(x,t)}{\partial t} \tag{A1}$$

and the initial equilibrium hole density distribution is a straight line of the form

$$P_{ss}(x) = P_c(0) + \left[P_E(0) - P_c(0)\right] \frac{x}{W} \tag{A2}$$

where according to (5)

$$I_{c\ sat} = q D_p \frac{P_E(0) - P_c(0)}{W} \tag{A3}$$

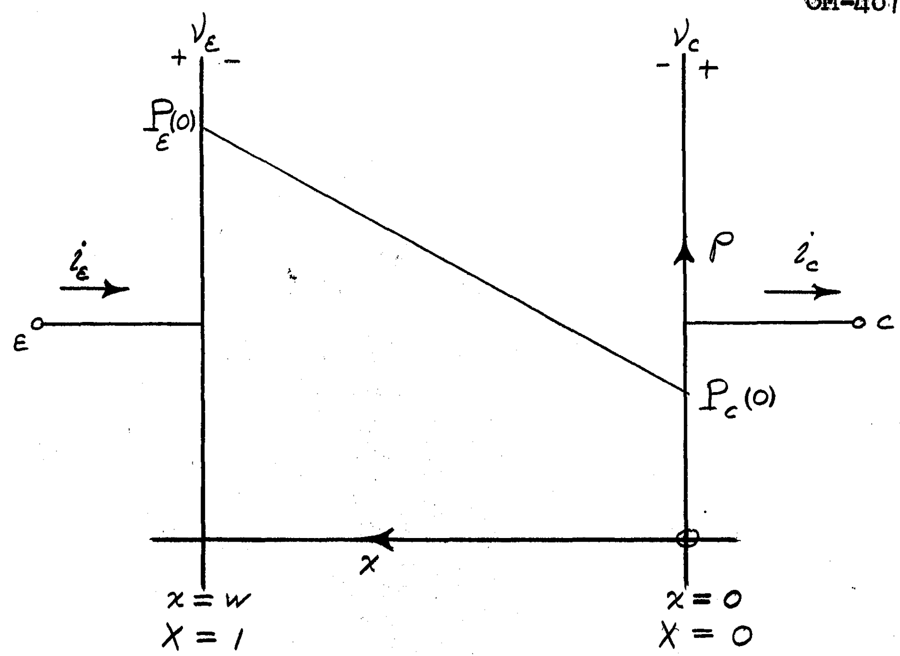

Fig. A1

Initial Equilibrium Hole Density Distribution in the Base

At time, t = 0, the switch, S_w, is thrown, instantaneously, to position 2. The positive voltage, V_2, is, thus, applied to the base and is assumed to be sufficiently large ($V_2 >> V_{cc}$) that both the emitter and collector junction are back biased at time $t = 0^+$,* i.e.,

$$v_\varepsilon \, , \, v_c < 0 \qquad (t > 0) \qquad \text{(A4)}$$

From (A4) and (4), the boundary conditions at the emitter and collector are, therefore, seen to be given by

$$\rho_\varepsilon = \rho_c = 0 \qquad (t > 0) \qquad \text{(A5)}$$

* See footnote on page 25

The initial condition of the hole density distribution, $p_{(x,0)}$, is the same as the equilibrium distribution at $t < 0$, given by (A2), since the hole density in the base can not change instantaneously except at the emitter and collector junction boundaries. Thus,

$$P(x,0) = P_c(0) + \left[P_E(0) - P_c(0) \right] \frac{x}{W} \qquad \text{(A6)}$$

For convenience in plotting the solution of the hole density distribution in the base, the diffusion equation is solved in its normalized form,

$$\frac{\partial^2 P(X,T)}{\partial X^2} = \frac{\partial P(X,T)}{\partial T} \qquad \text{(A7)}$$

$$\text{where} \quad X = \frac{x}{W}, \qquad T = \frac{t}{W^2/D_p}$$

The complete solution for $p(x,t)$, obtained by solving (A7) for the boundary and initial conditions (A5) and (A6) respectively, using the method of separation of variables, is found to be of the form

$$P(X,T) = \frac{2}{\pi} P_E(0) \sum_{n=1}^{\infty} \frac{1}{n} \left[\frac{P_c(0)}{P_E(0)} - (-1)^n \right] \left[\sin(n\pi X) \right] e^{-n^2\pi^2 T} \qquad \text{(A8)}$$

Equation (A8) can be plotted in a more convenient manner if it is expressed in the form

$$P_{norm} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left[S^{o} - (-1)^{n} \right] \left[\sin (n \pi X) \right] e^{-n^2 \pi^2 T}$$

$$\text{where} \quad P_{norm} = \frac{P(X,T)}{P_{\varepsilon}(0)} \qquad \text{(A9)}$$

$$S^{o} = \frac{P_{c}(0)}{P_{\varepsilon}(0)}$$

P_{norm} represents the hole density in the base as a function of time and position normalized with respect to the initial hole density at the emitter junction, $P_{\varepsilon}(0)$. Since $0 \leqslant P(X,T) \leqslant P_{\varepsilon}(0)$, P_{norm} always has a value which lies in the range, $0 \leqslant P_{norm} \leqslant 1$. S^{o} is the ratio of the initial hole density at the collector to the initial hole density at the emitter. When the transistor is not saturated $P_c(0) = 0$ and $S^{o} = 0$. The maximum saturation of the transistor occurs when $P_c(0) = P_{\varepsilon}(0)$ and $S^{o} = 1$. Thus, S^{o} can be considered as representing the degree of saturation of the transistor. Fig. A2 and A3 each show a plot of P_{norm} vs X, with time as a parameter, for two different degrees of saturation, $S^{o} = 0$ and $S^{o} = 0.5$, respectively.

The analytic expressions for the emitter and collector currents are obtained by substituting the gradient of $p_{(x,t)}$, evaluated at the emitter and collector junction boundaries, respectively, into (5). In this case, where the hole density is expressed in terms of x normalized with respect to w, a normalized form of (5) must be used. Thus,

$$\dot{i}_{\varepsilon,c} = \frac{q D_p}{W} \left. \frac{\partial P(X,T)}{\partial X} \right|_{1,0} \tag{A10}$$

The gradient of the hole density as obtained from (A8) can be written as

$$\frac{\partial P(X,T)}{\partial X} = 2 P_\varepsilon(0) \sum_{n=1}^{\infty} \left[S^{\circ} - (-1)^n \right] \left[\cos(n\pi X) \right] e^{-n^2 \pi^2 T} \tag{A11}$$

It follows from (A10) that the hole current at any point in the base, $\dot{i}_X$, is of ; the form

$$\dot{i}_X = 2 q D_p \frac{P_\varepsilon(0)}{W} \sum_{n=1}^{\infty} \left[S^{\circ} - (-1)^n \right] \left[\cos(n\pi X) \right] e^{-n^2 \pi^2 T} \tag{A12}$$

Again for convenience in plotting, the expression for the hole current, $\dot{i}_X$ can be normalized by defining a normalized current, $\dot{i}_{X\ norm}$, such that

$$\dot{i}_{X\ norm} = \frac{\dot{i}_X}{q D_p \left(P_\varepsilon(0)/W \right)} \tag{A13}$$

It can be seen from (A3) that the term in the denominator of (A13) represents the maximum amount of equilibrium hole current that can flow anywhere in the base for a given value of $P_\varepsilon(0)$.

From (A10), (A12) and (A13), the normalized expression for the emitter and collector currents are seen to be of the forms

$$-\dot{i}_{\varepsilon\ norm} = 2 \sum_{n=1}^{\infty} \left[1 - (-1)^n S^{\circ} \right] e^{-n^2 \pi^2 T} \tag{A14}$$

$$\dot{i}_{c\ norm} = 2 \sum_{n=1}^{\infty} \left[S^{\circ} - (-1)^n \right] e^{-n^2 \pi^2 T} \tag{A15}$$

where the current is defined to be positive when it "flows" in the positive direction of X. Fig. A4 and A5 show plots of $-i_{e\ norm}$ vs T and $i_{c\ norm}$ vs T, respectively, for various values of the parameter S^{o}.

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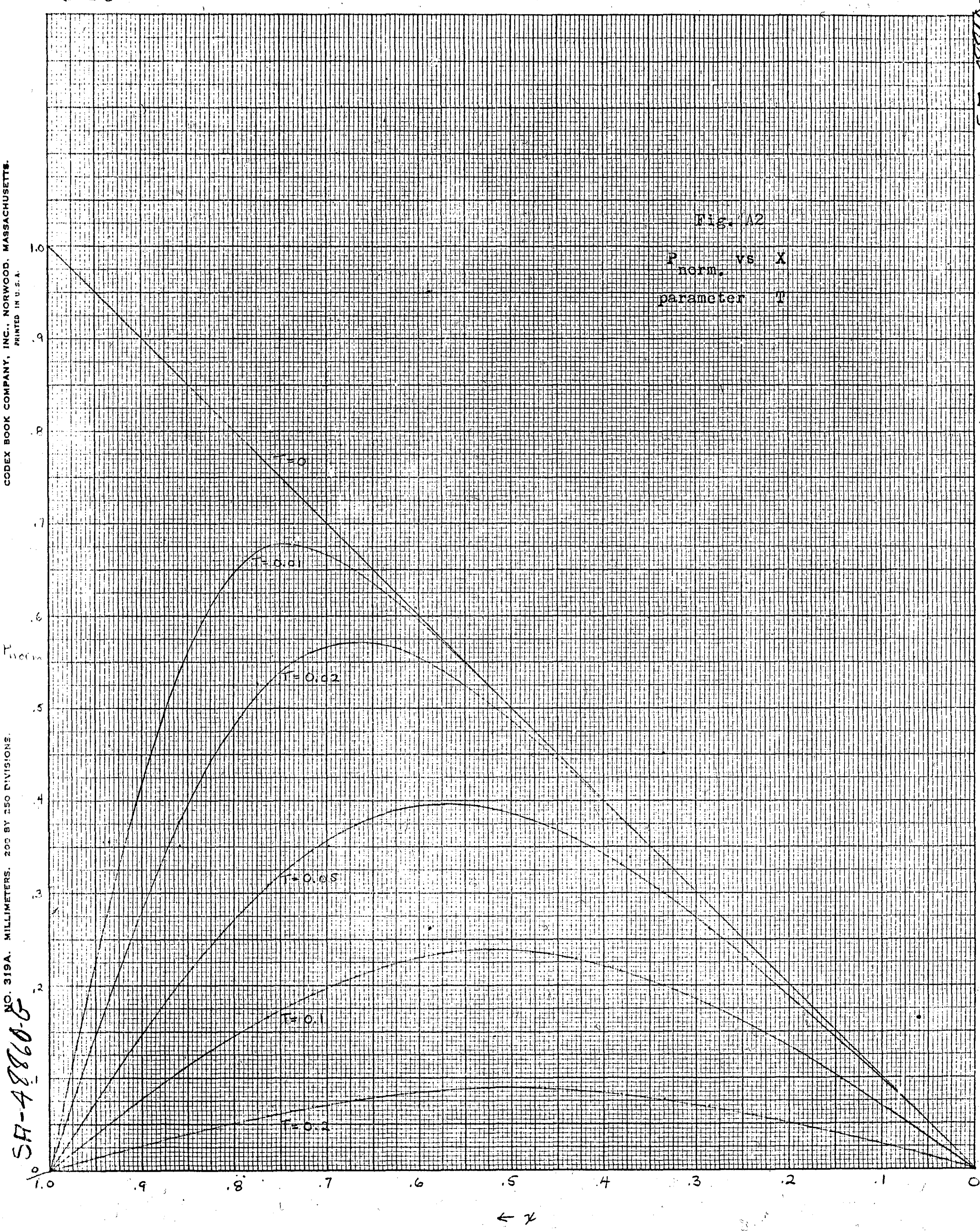
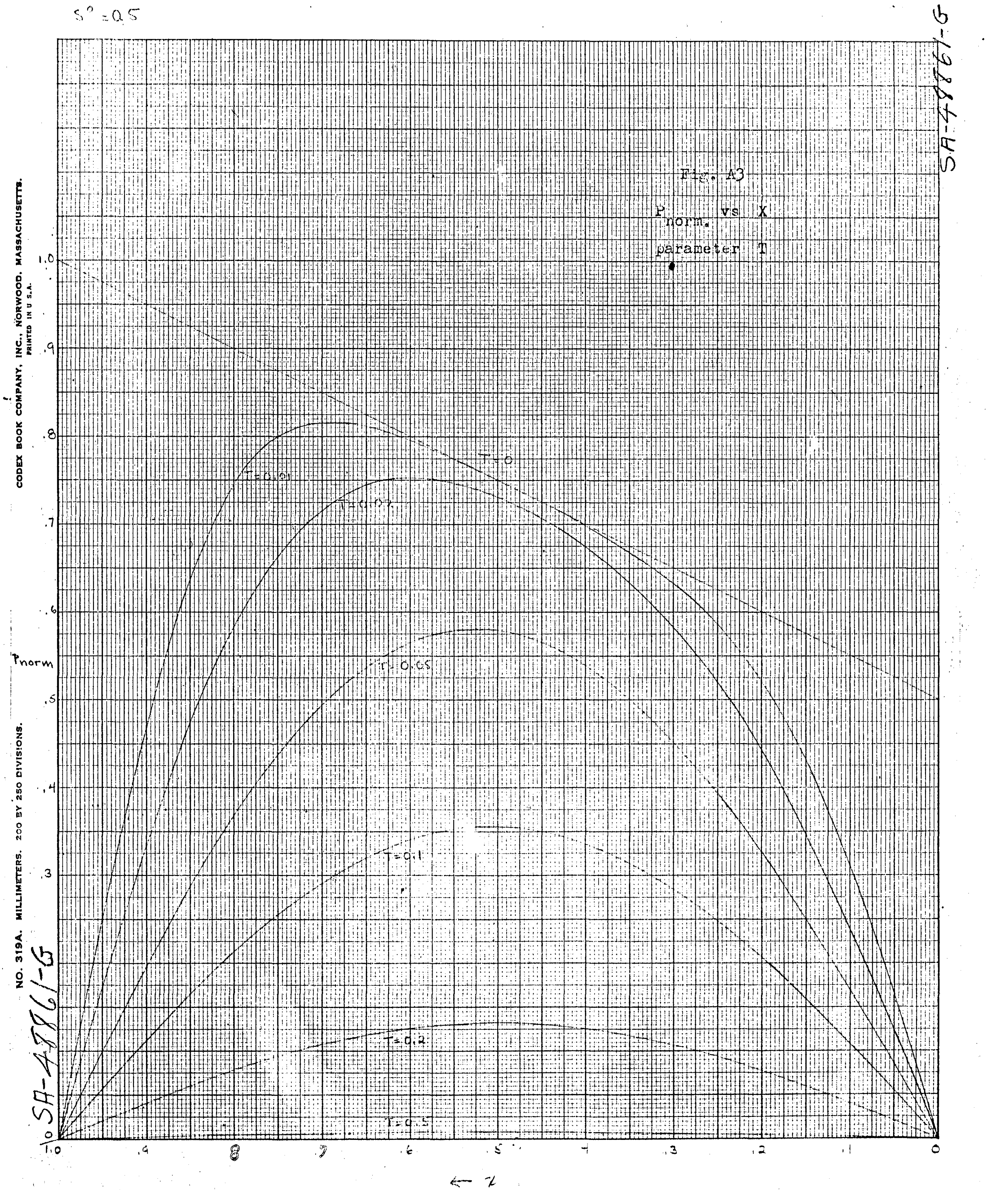
$S^{c} = 0$





S° = 0.5
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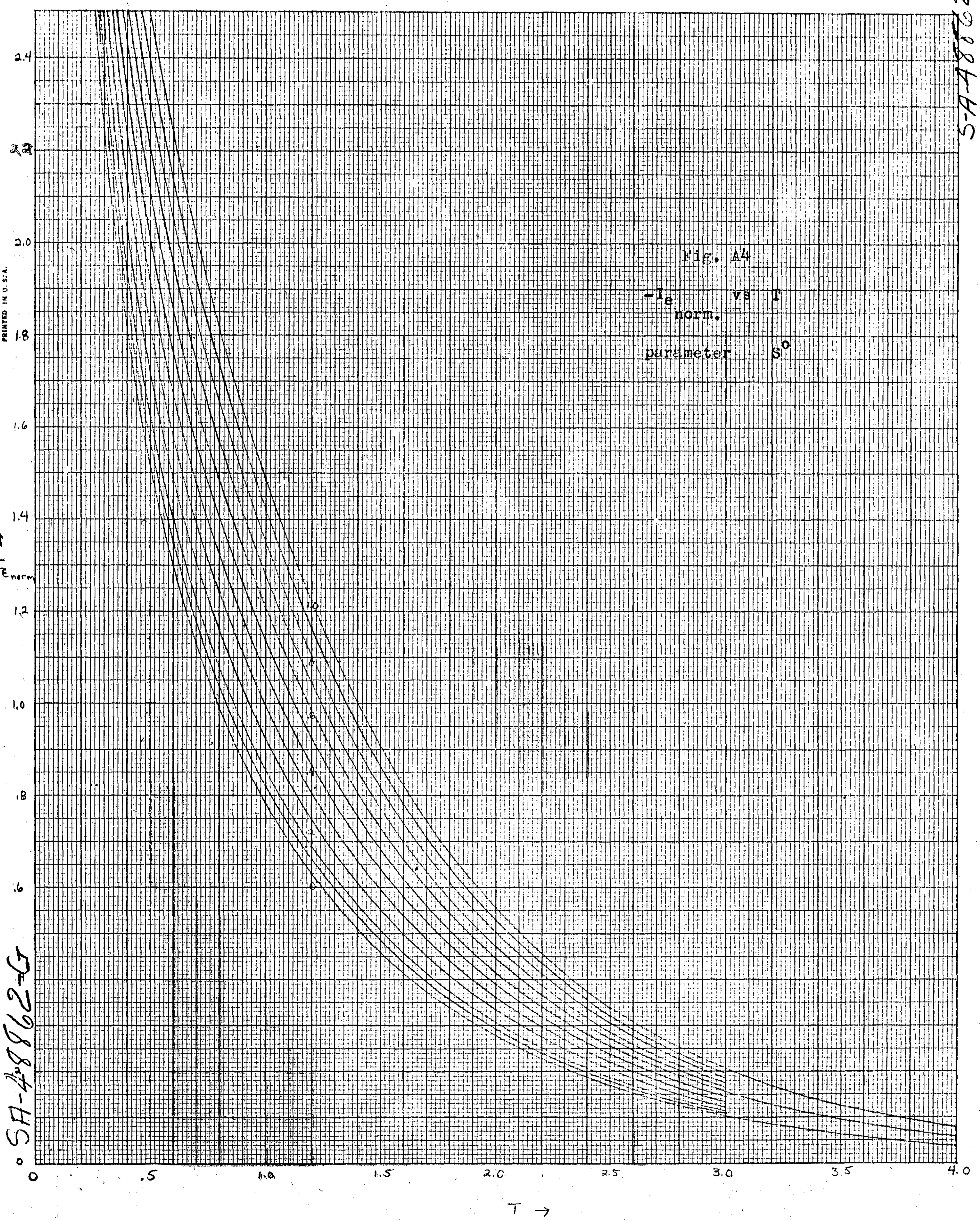
Fig. A3
P norm. vs X
parameter T
CODEX BOOK COMPANY, INC., NORWOOD, MASSACHUSETTS. PRINTED IN U.S.A.
NO. 319A. MILLIMETERS. 200 BY 250 DIVISIONS.
T=0
T=0.01
T=0.02
T=0.05
T=0.1
T=0.2
T=0.5
Pnorm
1.0
.9
.8
.7
.6
.5
.4
.3
0
.9
.8
.7
.6
.5
.4
.3
.2
.1
← x



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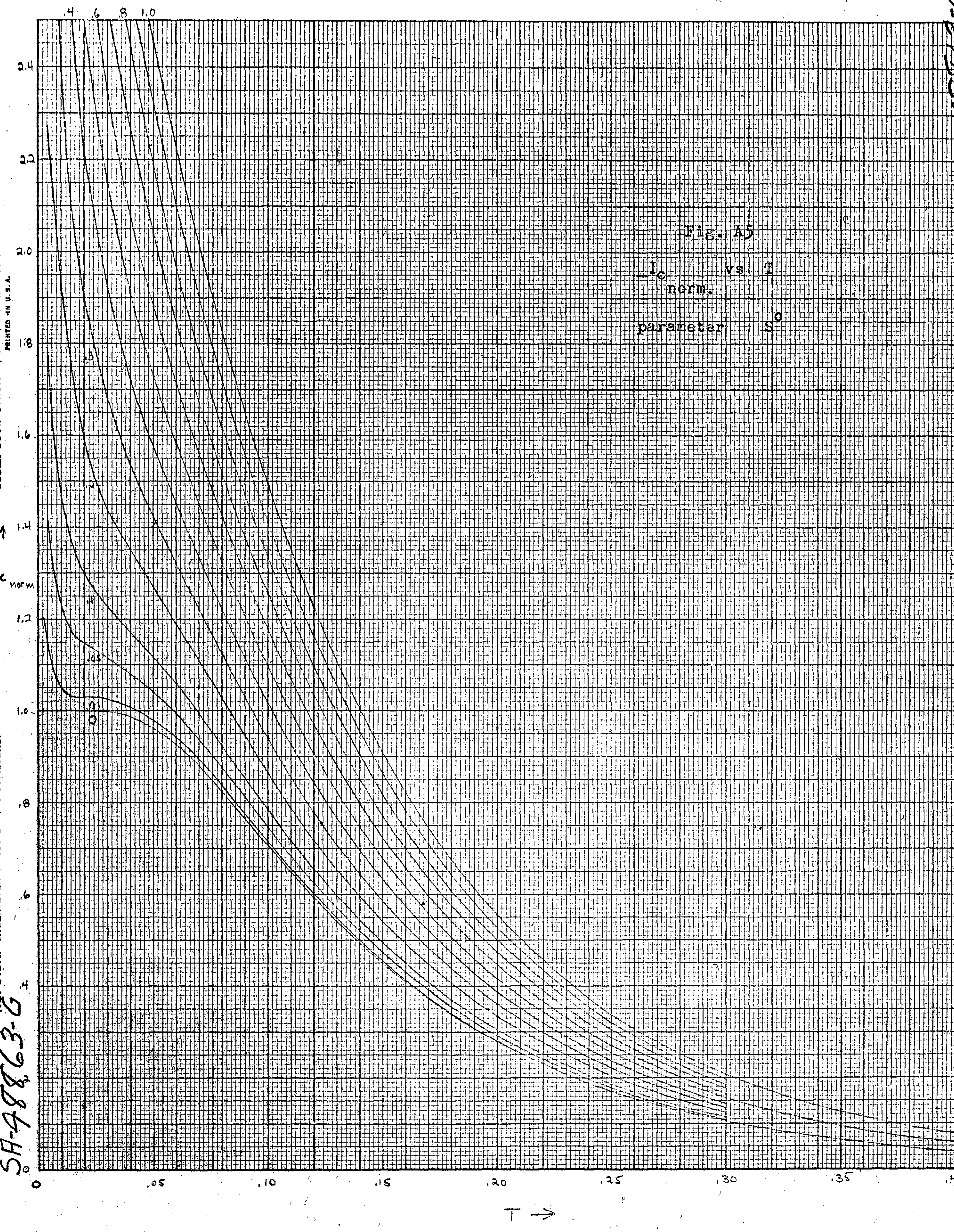
.4
.6
.8
1.0
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
0
.05
.10
.15
.20
.25
.30
.35
.40
T →
$H_{c\,norm}$ ↑
Fig. A5
H_c norm. vs T
parameter S°

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